Shape reconstruction of defects in multi-frequency eddy current testing using the level set and Tellegen's adjoint method

Piotr Putek [a], Marian Slodička [a], Roger van Keer [a], and Konstatnty M. Gawrylczyk [b]

[a] NaM^2 Research Group, Department of Mathematical Analysis, Ghent University, B-9000 Ghent, Belgium
Piotr.Putek@UGent.be

Department of Electrotechnology and Diagnostic, West Pomeranian University of Technology,
70-313 Szczecin, Poland
Konstanty.Gawrylczyk@zut.edu.pl

Abstract — An inverse problem of defects reconstruction in a metallic plate from the multi-frequency eddy current data is investigated. The application of a multi-frequency excitation for a different position of a sensor versus the region of interest enables to obtain more information on the inhomogeneity included in the measurement signal. In order to avoid the parameterization of a reconstructed shape, the more universal method, especially from a topological viewpoint, might be used. Developing such a numerical technique that can handle topology changes is very important for the defect reconstruction. Therefore, during an iterative optimization based on the regularized Gauss-Newton algorithm, the shape of a crack and its evolution was represented by the level sets of a continuous function φ called the level set function. For the purpose of the gradient calculation, the numerically very effective Tellegen's method of an adjoint model was applied. In this way, the accuracy of an applied inverse methodology as well as its robustness against an experimental noise have been improved. Numerical tests show the effectiveness of the algorithm for single inhomogeneity as well as in the case of multiple anomalies in a tested object. Finally, the application of both approaches to shape reconstruction by defect parameterization and level sets representation is presented.

I. INTRODUCTION

Non-destructive evaluation of conductive objects using the Eddy Current Testing (ECT) method has found a wide application in various fields of industry such as energy, automotive, marine, aeronautic, and manufacturing. From a practical viewpoint, especially the inverse problem of the crack and defects detection like metal-loss regions produced by corrosion, stress, fatigue seems to be a very important issue. Therefore, in this paper we develop the level sets-based methodology to solve the inverse problem of a material structure recognition arising from the Eddy Current Tomography. We investigate the case where the spatial conductivity distribution $\sigma(\mathbf{x})$ is a piecewise constant function represented by level set functions, with a possibility that the conductivity values inside the defects are treated also as unknown. Since the inverse problem of defect reconstruction is highly ill-posed, we introduce the regularization techniques such as the total variation norm of $\sigma(\mathbf{x})$ and Tikhonov stabilizing term β.

To knowledge of the authors, this powerful and versatile method of topology optimization based on sets level approach has not, until now, been applied in the Finite Element (FE) application for the purpose of defects reconstruction from the ECT signal.

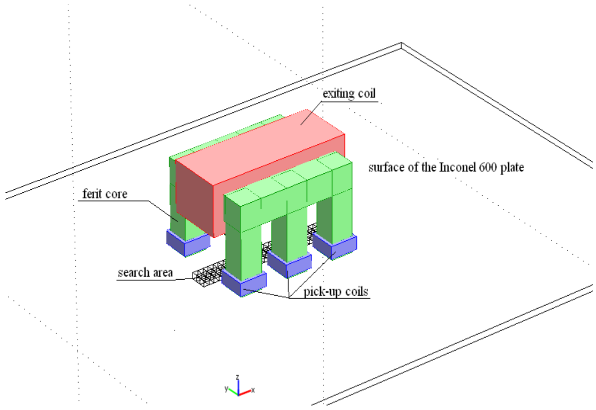

Fig. 1. The ECT system for crack reconstruction.

II. FORWARD PROBLEM IN ECT

In the case of ECT technique, the measurement of voltage or equivalently impedance of a pick-up coil at the different positions and for considerably varied frequency of the excitation current allows to reveal the information about the inhomogeneity such as its type, dimension and location. The used multi-frequency technique is especially useful when the defects inside of a thick object or on the reverse side versus applied sensor are considered. The model of analyzed ETC setup with the matrix type sensor [1] is depicted in Fig.1.

A. *Mathematical model of ECT*

In this paper, we assume that for the frequency range of the excitation current, the skin depth is sufficiently small compared to the depth or curvature of the conducting region. Therefore, the surface impedance boundary condition (SIBC) can be applied [2]. In consequence, only Γ_2 the surface of a conducting object with the conductivity satisfying $\sigma(\mathbf{x}) \geq \sigma_0 \geq 0$ can be discretized. In such case, the field distribution inside V a bounded domain in R^3 with C^1 boundary ∂V is governed by the 3D scalar Poisson equation. Then, the magnetic potential $u(\mathbf{x})$ inside V satisfies [3]

$$\nabla \cdot (\nabla u) = f \text{ in } V, \tag{1}$$

$$f = \nabla \cdot \mathbf{T}, \mathbf{J} = \nabla \times \mathbf{T}, \tag{2}$$

$$\frac{\partial u}{\partial n} = \nabla \cdot (\alpha \nabla u) \text{ on } \Gamma_2. \tag{3}$$

Here, $\mathbf{T}$ means the electrical vector potential while α stands for the converse of the propagation constant $k^2 = j\omega\mu\sigma$.

B. *Level set representation*

For the simplicity of description, we assume that Γ_2 consists only of two different materials Ω_1 and Ω_2 with the electrical conductivities σ_1 and σ_2, respectively. Then, the interface between them Γ can be represented by a zero level set of φ. We take φ in such way that $\Omega_1=\{\mathbf{x} \in \Gamma_2 \mid \varphi(\mathbf{x}) > 0\}$ and $\Omega_2=\{\mathbf{x} \in \Gamma_2 \mid \varphi(\mathbf{x}) < 0\}$. Finally, $\sigma(\mathbf{x})$ can be represented by

$$\sigma(\mathbf{x}) = \sigma_1 H(\varphi(\mathbf{x})) + \sigma_2\left(1 - H(\varphi(\mathbf{x}))\right) \quad \text{in } \Gamma_2, \qquad (4)$$

where $H(\mathbf{x})$ is the Heaviside function defined as $H(\mathbf{x}) = 1$ for $\mathbf{x} \geq 0$, and $H(\mathbf{x}) = 0$ when $\mathbf{x} > 0$. The derivative of $H(\mathrm{x})$ is the Dirac delta function. As a level set function φ we employ a signed distance function, thus $\varphi(\mathbf{x}) = d(\mathbf{x},\Gamma)$ when $\mathbf{x} \in \Omega_1$ and $\varphi(\mathbf{x}) = -d(\mathbf{x},\Gamma)$ for $\mathbf{x} \in \Omega_2$. The level set method, first proposed by [4] recently found a wide application in electrical engineering e.g. [5], [6] to the topology optimization problem.

III. Inverse Problem in Eddy Current Tomography

The inverse problem of defect reconstruction is provided by the minimization of a last-square functional of the data-model misfit $F(\varphi,\sigma_2)$ using level sets method in the Gauss-Newton algorithm. The gradient of the objective function is calculated basing on an adjoint model of Tellegen's method.

A. *Tellegen's adjoint method*

In the considered case, the sensitivity equation is given by [2]

$$\delta U = -j\omega \int_V \mathbf{J}_2 \delta(\nabla u_1) dV = k^2\sigma^{-2} \int_V \nabla u_1 \nabla u_2 \delta\sigma dV. \qquad (5)$$

Here, U means the voltage, the subscripts *1* and *2* refer to the original model defined by (1)-(3) and the adjoint model. The letter is based on (1) with (3) and (2) satisfying $\mathbf{J}_2 S_c = 1$ in the area of a pick up coil S_c. Finally, the sensitivity of the voltage in respect to the conductivity in e-th finite element, which lies in the plate, takes the form

$$\nabla_\sigma U = 0.5k\sigma^{-2} \int_{S^e} \nabla u_1 \nabla u_2 dS^e, \qquad (6)$$

where S^e stands for the field of the considered element. For a detailed description, see [3], [7].

B. *Incorporating level sets into Gauss-Newton algorithm*

The gradient of the objective function F with respect to σ, φ, σ_2 takes the form

$$\nabla_\sigma F = \sum_i \nabla_U F_i \cdot \nabla_\sigma U_i - \beta\nabla\cdot\left(\nabla\sigma\cdot|\nabla\sigma|^{-1}\right), \qquad (7)$$

$$\nabla_\sigma F = \nabla_\sigma F(\sigma_1 - \sigma_2)H'(\varphi), \qquad (8)$$

$$\nabla_{\sigma_2} F = \int_{\Gamma_2} \nabla_\sigma F(1 - H(\varphi))dx. \qquad (9)$$

The above formulas from (6) to (9) allow to easily combine the level set methods to any step descent algorithm.

IV. Results of Shape Reconstruction by Defect Parameterization Approach

The objective of previous research conducted in [3] was to reconstruct the conductivity of a surface crack in a plate made of Inconel 600 based on the parameterization approach. The search area consisted of 168 finite elements. The FEM-SIBC simulation of ETC measurement was carried out using following parameters: $\boldsymbol{J}$ = 1MA/m^2, 5 number of exciting harmonic signal from 75 kHz to 200 kHz with 30 dB noise, 25 movements of probe along $0\boldsymbol{x}$ axis with Δ_k = 0.5 mm. The applied mesh consists of NE = 4800 finite elements, ND = 9471 nodes. Results of the identification process by means of the Gauss-Newton algorithm with TSVD are shown in Fig.2.

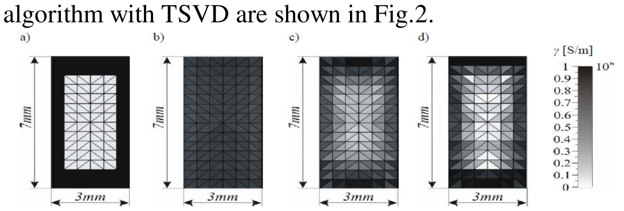

Fig. 2. Course of the defect identification in FEM-SIBC model using noised syntactical data: a) the assumed conductivity distribution, b - d) the identified conductivity distribution after 3th, 5th and 9th iteration [3].

V. Conclusion

The application of the level set-based algorithm to the ECT problem enables to recover sharp interfaces without the necessity of explicit shape parameterization, also if noise is present in the synthetic data. For the purpose of the gradient calculation, the efficient method of Tellegen's adjoint model is applied. This allows to reduce the computational complexity of defects identification procedures.

VI. References

[1] T. Chady, M. Enokizono, T. Tokada, Y. Tsuchida, R. Sikora, "A family of matrix type sensors for Detection of Slight Flaws in conducting Plates," *IEEE Trans. on Magn.*, vol. 35, no. 5, pp. 1093-1096, 1999.

[2] S. Barmada , L. Di Rienzo, N. Ida and S. Yuferev, "The use of surface impedance boundary conditions in time domain problems: numerical and experimental validation," *ACES Journal,* vol. 19, no. 2, pp. 76-83, 2004.

[3] K.M. Gawrylczyk, P. Putek. "Multi-Frequency Sensitivity Analysis of 3D Models Utilizing Impedance Boundary Condition with Scalar Magnetic Potential, " *IOS Press Advanced Computer Techniques in Applied Electromagnetic*, vol. 30, pp. 64-67, 2008.

[4] S.J. Osher, J.A. Sethian, "Fronts propagating with curvature dependent speed: algorithms based on Hamilton-Jacobi formulations,"*J. Comput. Phys.* vol. 79, pp. 12-49, 1988.

[5] Y.S. Kim; I. H. Park, "Topology Optimization of Rotor in Synchronous Reluctance Motor Using Level Set Method and Shape Design Sensitivity," *IEEE Trans. on Magn.*, vol.20, no.3, pp. 1093-1096, 2010.

[6] N. Irishina, M. Moscoso, O. Dorn, "Microwave Imaging for Early Breast Cancer Detection Using a Shape-based Strategy," *IEEE Transactions on Biomedical Engineering*, vol.56, no.4, pp.1143-1153, 2009.

[7] D.N. Dyck, D.A. Lowther, "A Method of Computing the Sensitivity of Electromagnetic Quantities to Changes in Material and Sources," *IEEE Trans. on Magn.*, vol.30, no 5, pp. 3415-3418, 1994.